Enrollment No. _____

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous)
Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION APRIL – 2017

M.Sc. Mathematics

16PMTCC09 – THEORY OF PARTIAL DIFFERENTIAL EQUATIONS

Duration of Exam – 3 hrs ***Semester – II*** ***Max. Marks – 70***

***Part A** (5x2= 10 marks)*

Answer **ALL** questions

1. Write the General Solution of $(D+2D'-3)(D^2+D')z=0$.
2. Classify the P.D.E. $xyr-(x^2-y^2)s-xyt+py-qx=2(x^2-y^2)$.
3. Solve $xy^2s=1-2x^2y$
4. Solve $t=y^2e^x$
5. Write the general solution of heat equation.

***Part B** (5X5 = 25 marks)*

Answer **ALL** questions

6a. Obtain the Complementary Function of an irreducible non-homogeneous Linear Partial Differential Equation $F(D,D')z=0$.

OR

6b. Prove that $\frac{1}{F(D,D')}e^{ax+by}\¤(x,y)=e^{ax+by}\frac{1}{F(D+a,D'+b)}\¤(x,y)$.

7a. Solve $\left(D^3+D^2D'-DD'^2-D'^3\right)z=e^y\cos 2x$ by General Method.

OR

7b. Solve $\left(D^2-6DD'+9D'^2\right)z=12x^2+36xy$.

8a. Solve $\frac{\partial^2 u}{\partial x^2}=20\frac{\partial u}{\partial y}$ by separation of variables.

OR

8b. Solve $t-q-\frac{1}{x}\left(\frac{1}{x}-1\right)z=xy^2-x^2y^2+2x^3y-2x^3$

9a. A string is stretched and fastened at two points 'l' apart. Motion is started by displacing the string in the form $y=a\sin\left(\frac{\®x}{l}\right)$. The initial velocity is zero. Find the displacement at any point on the string at time t.

OR

9b. Solve $r-\frac{y}{x}s=15xy^2$

10a. Find the temperature $u(x,t)$ in an insulated copper bar of 80 meters long. The initial temperature is $200\sin\left(\frac{®x}{80}\right)$. The ends are kept at $0^{\circ}c$. For copper $c^2 = 1.158$

OR

10b. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for the given boundary conditions as

$u(o,y) = 0,\ u(l,y) = 0,\ u(x,0) = 0,\ u(x,a) = \sin\left(\frac{n®x}{l}\right)$

<u>Part C</u> (5X7 = 35 marks)

Answer **<u>ALL</u>** questions

11a. Show that the Complementary Function of the Partial Differential Equation $(aD + bD' + c)(aD + bD' + c)z = f(x,y)$ is $e^{-\frac{c}{a}x}\left[¤_1(bx - ay) + x¤_2(bx - ay)\right]$, if $a \neq 0$ where $¤_1$ and $¤_2$ are arbitrary functions.

OR

11b. Describe Monge's Method for solving $Rr + Ss + Tt = V$.

12a. Obtain the canonical form of the P.D.E.
$x^2(y-1)r - x(y^2-1)s + y(y-1)t + xyp - q = 0$.

OR

12b. Obtain the canonical form of the P.D.E. $r + 2xs + x^2 t = 0$.

13a. Solve $r + a^2 t = 0$ by Monge's Method.

OR

13b. Solve $q^2 r - 2pqs + p^2 t = 0$ by Monge's Method.

14a. Explain the solution of Laplace equation in spherical coordinates.

OR

14b. Find the solution $u(x,t)$ for the partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ by separation of variables.

15a. Derive D'Alembert's solution of wave equation.

OR

15b. Describe heat equation problem. Also explain the mathematical model of heat equation.